HYDROMAGNETIC EQUATIONS OF A RAREFIED PLASMA

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The general hydromagnetic equations are obtained for a collisionless plasma, taking account of "magnetic viscosity" and thermal conductivity, when (1.2) holds. These equations are not closed since they contain the fourth moments. By computing these moments, for example, by Grad's method, we can close the system of equations. A system of equations in two-dimensional theory is also given.

The form of the macroscopic equations of a rarefied plasma in a magnetic field depends strongly on the procedure for ordering the physical variables. The Chew, Goldberger, Low (CGL) [1-4] ordering, the ordering of the theory of finite Larmor radius [5-7], the low-density ordering [8], and others are known. The appropriate expansion of the distribution function is frequently used to obtain the hydromagnetic equations [1,3,7,9]. But a simpler method is to use the infinite chain of moment equations directly [4]. Grad's modified moment method [10-12], which, however, has a number of deficiencies, is very close to this method.

1. A closed series of macroscopic equations was obtained in [3] for the CGL ordering

$$\frac{\omega}{\Omega} \sim \frac{a}{L} \sim \frac{\omega}{\omega_p} \sim \frac{\lambda_D}{L} \sim \varepsilon \ll 1 \tag{1.1}$$

Here Ω, ω_p are the cyclotron and plasma frequencies; ω is a typical frequency of macrochanges; a, λ_D are the Larmor radius and Debye length; L is a typical macrolength. The order (1.1) implies that $\Omega \sim \omega_p$. But in many practically important cases $\Omega \ll \omega_p$. The aim of this paper is to deduce the hydromagnetic equations of a collisionless plasma for the ordering

$$\omega / \Omega \sim a / L \sim \varepsilon \ll 1, \quad \Omega / \omega_p \sim \lambda_D / a \sim \mu \ll 1 \tag{1.2}$$

The required equations are obtained directly from the infinite chain of moment equations by expanding all the variables in series in the small parameters ε and μ, which are assumed to be independent.

A similar two-parameter expansion was used in [9], where the equations of the lowest approximation in ε and β (β is the ratio of the material pressure to the magnetic pressure) were obtained. In contradistinction to this expansion, (1.2) makes it possible, for example, to investigate the stability of the plasma as a function of β. We know [1,2] that the CGL expansion does not lead to a closed system of equations, since these equations contain the vector for the heat flux (the third moment) along the magnetic field. To determine the longitudinal heat flux requires knowledge of the fourth moments. To determine the latter we need the fifth moments, etc. This difficulty is usually eliminated by neglecting the longitudinal heat fluxes [3, 9] or by using so-called two-dimensional theory [3]. In these cases a closed system of equations is obtained, suitable for physical applications.

2. As the initial system of equations we take the chain of moment equations obtained in a standard manner from the Vlasov kinetic equation and the system of Maxwell's equations (cf. [4]). In these equations the independent variables are (in standard notation)

$$\mathbf{E}, \mathbf{B}, \rho, \mathbf{u}, \boldsymbol{P}, \boldsymbol{Q}, \boldsymbol{R}, \ldots$$

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In place of these it is convenient to introduce the following variables:

$$\mathbf{V},\ E_\parallel,\ B,\ \mathbf{e}_1,\ \rho,\ \mathbf{u}_\perp,\ u_\parallel,\ p_\perp,\ p_\parallel,\ \sigma_{\alpha\beta},\ Q_{\alpha\beta\gamma},\ \ldots \tag{2.1}$$

Here

$$\mathbf{e}_1 \equiv \mathbf{B}/B,\ A_\parallel = \mathbf{A}\cdot\mathbf{e}_1,\ \mathbf{A}_\perp = \mathbf{A} - A_\parallel \mathbf{e}_1$$

$\mathbf{V} = (c/B)[\mathbf{E}_\perp \mathbf{e}_1]$ is the velocity of electric drift; $p_\parallel = \mathbf{e}_1\mathbf{e}_1 : P \equiv P_{11}$ is the "longitudinal" pressure; $p_\perp = \frac{1}{2}(I - \mathbf{e}_1\mathbf{e}_1) : P \equiv \frac{1}{2}(P_{22} + P_{33})$ is the "transverse" pressure; $\mathbf{I}$ is the unit tensor; $\sigma_{\alpha\beta}$ is the viscous stress tensor with the properties $\sigma_{11} = 0$, $\mathrm{Sp}\,\boldsymbol{\sigma} = 0$, $Q_{\alpha\beta\gamma} = \mathbf{e}_\alpha\mathbf{e}_\beta : (\mathbf{e}_\gamma \cdot \mathbf{Q})$ is the projection of the heat flux tensor $\mathbf{Q}$ on the coordinate axes of the local coordinate system formed by a right-handed orthogonal triplet of unit vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ (α, β, γ take the values 1, 2, 3).

In terms of the variables (2.1) the original system of equations takes the form

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho(\mathbf{u}_\perp + u_\parallel \mathbf{e}_1) = 0 \tag{2.2}$$

$$\left(\frac{d\mathbf{u}}{dt}\right)_\parallel + \frac{1}{\rho}(\nabla\cdot\boldsymbol{P})_\parallel - \frac{e}{m}E_\parallel = 0 \tag{2.3}$$

$$\mathbf{u}_\perp - \mathbf{V} = \frac{1}{\Omega}\left[\mathbf{e}_1\left(\frac{d\mathbf{u}}{dt} + \frac{1}{\rho}\nabla\cdot\boldsymbol{P}\right)\right] \tag{2.4}$$

$$\frac{d\boldsymbol{P}}{dt} + \boldsymbol{P}\nabla\cdot\mathbf{u} + \nabla\cdot\boldsymbol{Q} + [\boldsymbol{P}\cdot\nabla\mathbf{u}]^s = \Omega\,[\boldsymbol{P}\times\mathbf{e}_1]^s \tag{2.5}$$

$$\frac{dp_\perp}{dt} = -2p_\perp\nabla\cdot\mathbf{u} + p_\perp\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{u} - \boldsymbol{\sigma} : \nabla\mathbf{u} - \nabla\cdot\mathbf{q} + \frac{1}{2}\mathbf{e}_1\mathbf{e}_1 : (\nabla\cdot\boldsymbol{Q}) + \mathbf{e}_1(\mathbf{e}_1\cdot\boldsymbol{\sigma}) : \nabla\mathbf{u} - \frac{1}{2}\boldsymbol{\sigma} : \frac{d}{dt}\mathbf{e}_1\mathbf{e}_1 \tag{2.6}$$

$$\frac{dp_\parallel}{dt} = -p_\parallel\nabla\cdot\mathbf{u} - 2p_\parallel\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{u} - 2\mathbf{e}_1(\mathbf{e}_1\cdot\boldsymbol{\sigma}) : \nabla\mathbf{u} - \mathbf{e}_1\mathbf{e}_1 : (\nabla\cdot\boldsymbol{Q}) + \boldsymbol{\sigma} : \frac{d}{dt}\mathbf{e}_1\mathbf{e}_1 \tag{2.7}$$

$$\frac{d\boldsymbol{Q}}{dt} + \boldsymbol{Q}\hat{\nabla}\cdot\mathbf{u} + \nabla\cdot\boldsymbol{R} + [\boldsymbol{Q}\cdot\nabla\mathbf{u}]^s - \frac{1}{\rho}[\boldsymbol{P}\nabla\cdot\boldsymbol{P}]^s = \Omega\,[\boldsymbol{Q}\times\mathbf{e}_1]^s \tag{2.8}$$

$$\frac{d}{dt}2q_\parallel^\parallel + 2q_\parallel^\parallel\nabla\cdot\mathbf{u} + \mathbf{e}_1\mathbf{e}_1 : (\mathbf{e}_1\nabla : \boldsymbol{R}) + 6q_\parallel^\parallel\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{u} = \frac{3p_\parallel}{\rho}\{\mathbf{e}_1\cdot\nabla p_\parallel + (p_\parallel - p_\perp)\nabla\cdot\mathbf{e}_1\} \tag{2.9}$$

$$\frac{d}{dt}2q_\parallel^\perp + 4q_\parallel^\perp\nabla\cdot\mathbf{u} + (\boldsymbol{I} - \mathbf{e}_1\mathbf{e}_1) : (\mathbf{e}_1\nabla : \boldsymbol{R}) = \frac{2p_\perp}{\rho}\{\mathbf{e}_1\cdot\nabla p_\parallel + (p_\parallel - p_\perp)\nabla\cdot\mathbf{e}_1\} \tag{2.10}$$

$$\frac{\partial B}{\partial t} = -\nabla\cdot B\mathbf{V} + B\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{V} - cE_\parallel\mathbf{e}_1\cdot\mathrm{rot}\,\mathbf{e}_1 \tag{2.11}$$

$$\frac{\partial\mathbf{e}_1}{\partial t} = -\mathbf{e}_1(\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{V}) + \mathbf{e}_1\cdot\nabla\mathbf{V} - \mathbf{V}\cdot\nabla\mathbf{e}_1 - \frac{c}{B}\{E_\parallel\,\mathrm{rot}_\perp\mathbf{e}_1 + [\nabla E_\parallel\mathbf{e}_1]\} \tag{2.12}$$

$$\frac{\partial E_\parallel}{\partial t} = -4\pi j_\parallel + cB\,\mathbf{e}_1\cdot\mathrm{rot}\,\mathbf{e}_1 + \frac{B}{c}\mathbf{V}\cdot[\mathbf{e}_1\{\mathbf{V}\cdot\nabla\mathbf{e}_1 - \mathbf{e}_1\cdot\nabla\mathbf{V}\}] - E_\parallel\mathbf{V}\mathbf{e}_1 : \nabla\mathbf{e}_1 + \mathbf{V}\cdot\nabla E_\parallel \tag{2.13}$$

$$\left(\frac{\partial\mathbf{V}}{\partial t}\right)_\perp = -\frac{c^2}{B}\nabla_\perp B + \left\{c^2 + \left(c\frac{E_\parallel}{B}\right)^2\right\}\mathbf{e}_1\cdot\nabla\mathbf{e}_1 + \frac{4\pi c}{B}[\mathbf{e}_1\mathbf{j}] - c^2\frac{E_\parallel}{B^2}\nabla_\perp E_\parallel$$

$$-\mathbf{V}\left(\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{V} - \frac{1}{B}\nabla\cdot B\mathbf{V} - \frac{cE_\parallel}{B}\mathbf{e}_1\cdot\mathrm{rot}\,\mathbf{e}_1\right) + \frac{cE_\parallel}{B}[\mathbf{e}_1\{\mathbf{e}_1\cdot\nabla\mathbf{V} - \mathbf{V}\cdot\nabla\mathbf{e}_1\}] \tag{2.14}$$

$$\nabla\cdot\mathbf{e}_1 + \mathbf{e}_1\cdot\nabla\ln B = 0 \tag{2.15}$$

$$E_\parallel\nabla\cdot\mathbf{e}_1 + \mathbf{e}_1\cdot\nabla E_\parallel + \frac{B}{c}(\mathbf{V}\cdot\mathrm{rot}\,\mathbf{e}_1 - \mathbf{e}_1\cdot\mathrm{rot}\,\mathbf{V}) + \frac{1}{c}[\mathbf{e}_1\mathbf{V}]\cdot\nabla B = 4\pi\sigma \tag{2.16}$$

In these equations we have omitted the subscript a which enumerates the kind of the particles,

$$\mathbf{j} = \sum_a e_a n_a \mathbf{u}_a,\quad \sigma = \sum_a e_a n_a,\quad \Omega = \frac{eB}{mc},\quad \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla,\quad \nabla_\perp = \nabla - \mathbf{e}_1\mathbf{e}_1\cdot\nabla$$

The abbreviation $[\ldots]^s$ denotes the sum of terms obtained by cyclic permutation of the subscripts; $\mathbf{ab:cd}$ is the double scalar product of the dyads $\mathbf{ab}$ and $\mathbf{cd}$; $\mathbf{q}^{\parallel}$ and $\mathbf{q}^{\perp}$ are the heat energy fluxes in the parallel and transverse directions

$$\mathbf{q}^{\parallel} = {}^1/_2 \boldsymbol{Q} : \mathbf{e}_1\mathbf{e}_1, \quad \mathbf{q}^{\perp} = {}^1/_2 \boldsymbol{Q} : (\boldsymbol{I} - \mathbf{e}_1\mathbf{e}_1), \quad \mathbf{q} = {}^1/_2 \boldsymbol{Q} : \boldsymbol{I}$$

If we now write the equations in dimensionless form using typical scales (length L, time T, thermal velocity v_T, etc.) and express all variables as expansions in a double series in ε and μ, we can obtain equations for the successive approximations.

3. In the zero-order approximation the tensors **P**, **Q**, **R** satisfy equations of the form

$$[\boldsymbol{A} \times \mathbf{e}_1]^s = 0 \tag{3.1}$$

It follows from these equations that in the zero-order approximation

$$\boldsymbol{P} = p_{\parallel}\mathbf{e}_1\mathbf{e}_1 + p_{\perp}(\boldsymbol{I} - \mathbf{e}_1\mathbf{e}_1), \quad \sigma = 0 \tag{3.2}$$

$$\boldsymbol{Q} = (2q_{\parallel}^{\parallel} - 3q_{\parallel}^{\perp})\,\mathbf{e}_1\mathbf{e}_1\mathbf{e}_1 + q_{\parallel}^{\perp}[\mathbf{e}_1\boldsymbol{I}]^s \tag{3.3}$$

$$\boldsymbol{R} = (R_1 - 6R_2 + {}^3/_4 R_3)\,\mathbf{e}_1\mathbf{e}_1\mathbf{e}_1\mathbf{e}_1 + (R_2 - {}^1/_4 R_3)[\mathbf{e}_1\mathbf{e}_1\boldsymbol{I}]^s + {}^1/_4 R_3\,[\boldsymbol{I}\boldsymbol{I}]^s \tag{3.4}$$

where $R_1 = m\,\langle c_{\parallel}^4\rangle$ $R_2 = (m/2)\langle c_{\parallel}^2 c_{\perp}^2\rangle$, $R_3 = (m/2)\,\langle c_{\perp}^4\rangle$, $\mathbf{c}$ is the random velocity of the particles, $\langle\ldots\rangle$ denotes statistical averaging.

Further, we see from (2.3) and (2.4) that in the zero-order approximation there is no electric field, $E_{\parallel} = 0$, while the transverse velocity of the particles coincides with the electrical drift velocity

$$\mathbf{u}_{\perp} = \mathbf{V} \tag{3.5}$$

To obtain the equation for the evolution of the longitudinal velocity $u_{\parallel}$ we have to consider the first approximation with respect to ε in Eq. (2.3)

$$\frac{\partial u_{\parallel}}{\partial t} + \mathbf{V}\cdot\nabla u_{\parallel} + u_{\parallel}\mathbf{e}_1\cdot\nabla u_{\parallel} - \mathbf{V}\cdot\frac{\partial \mathbf{e}_1}{\partial t} + \mathbf{e}_1\mathbf{V} : \nabla\mathbf{V} + u_{\parallel}\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{V}$$
$$+ \frac{1}{\rho}\{\mathbf{e}_1\cdot\nabla p_{\parallel} + (p_{\parallel} - p_{\perp})\nabla\cdot\mathbf{e}_1\} = \frac{e}{m}E_{\parallel}^{(1)} \tag{3.6}$$

Further, in the zero-order approximation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho(\mathbf{V} + u_{\parallel}\mathbf{e}_1) = 0 \tag{3.7}$$

$$\frac{d_0 p_{\perp}}{dt} = -2p_{\perp}\nabla\cdot\mathbf{u} + p_{\perp}\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{u} - 2q_{\parallel}^{\perp}\nabla\cdot\mathbf{e}_1 - \mathbf{e}_1\cdot\nabla q_{\parallel}^{\perp} \tag{3.8}$$

$$\frac{d_0 p_{\parallel}}{dt} = -p_{\parallel}\nabla\cdot\mathbf{u} - 2p_{\parallel}\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{u} - 2(q_{\parallel}^{\parallel} - q_{\parallel}^{\perp})\nabla\cdot\mathbf{e}_1 - 2\mathbf{e}_1\cdot\nabla q_{\parallel}^{\parallel} \tag{3.9}$$

$$\frac{d_0}{dt}2q_{\parallel}^{\parallel} + 2q_{\parallel}^{\parallel}(\nabla\cdot\mathbf{u} + 3\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{u}) + \mathbf{e}_1\cdot\nabla R_1 + (R_1 - 3R_2)\nabla\cdot\mathbf{e}_1 = \frac{3p_{\parallel}}{\rho}\{\mathbf{e}_1\cdot\nabla p_{\parallel} + (p_{\parallel} - p_{\perp})\nabla\cdot\mathbf{e}_1\} \tag{3.10}$$

$$\frac{d_0 2q_{\parallel}^{\perp}}{dt} + 4q_{\parallel}^{\perp}\nabla\cdot\mathbf{u} + 2\mathbf{e}_1\cdot\nabla R_2 + (4R_2 - R_3)\nabla\cdot\mathbf{e}_1 = \frac{2p_{\perp}}{\rho}\{\mathbf{e}_1\cdot\nabla p_{\parallel} + (p_{\parallel} - p_{\perp})\nabla\cdot\mathbf{e}_1\} \tag{3.11}$$

$$\frac{\partial B}{\partial t} = -\nabla\cdot B\,\mathbf{V} + B\,\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{V} \tag{3.12}$$

$$\frac{\partial\,\mathbf{e}_1}{\partial t} = -\mathbf{e}_1\mathbf{e}_1\mathbf{e}_1 : \nabla\mathbf{V} + \mathbf{e}_1\cdot\nabla\mathbf{V} - \mathbf{V}\cdot\nabla\mathbf{e}_1 \tag{3.13}$$

where

$$\frac{d_0}{dt} = \frac{\partial}{\partial t} + (\mathbf{V} + \mathbf{u}_{\parallel}\mathbf{e}_1)\cdot\nabla$$

To this system we have to add further equations for $E_{\parallel}^{(1)}$ and $\mathbf{V}$, which in general have to be obtained from (2.13) and (2.14). However, these equations contain first approximations in ε for the currents, to determine which we require equations for $\mathbf{u}$ in the first approximation with second approximations for the electric field and the currents, etc. This difficulty can be avoided (cf. [3]). It follows from the general equation for $\mathbf{u}$ that

$$E_{\parallel} \sum_a \frac{e_a^2 n_a}{m_a} = \frac{\partial j_{\parallel}}{\partial t} + \sum_a e_a u_{\parallel a} \nabla \cdot n_a \mathbf{u}_a - \mathbf{j} \cdot \frac{\partial \mathbf{e}_1}{\partial t} + \sum_a \frac{e_a}{m_a} \mathbf{e}_1 \nabla : \boldsymbol{P}_a \tag{3.14}$$

From this, in the first approximation in ε and the zero-order approximation in μ we have

$$E_{\parallel}^{(1,0)} \sum_a \frac{e_a^2 n_a^{(0,0)}}{m_a} = \sum_a e_a \left\{ u_{\parallel a} \nabla n_a \mathbf{u}_a + \frac{1}{m_a} [\mathbf{e}_1 \cdot \nabla p_{\parallel a} + (p_{\parallel a} - p_{\perp a}) \nabla \cdot \mathbf{e}_1] \right\}^{(0,0)} \tag{3.15}$$

since, as follows from (2.13) and (2.14),

$$j_{\parallel}^{(0,0)} = j_{\parallel}^{(1,0)} = j_{\parallel}^{(0,1)} = j_{\parallel}^{(0,2)} = j_{\parallel}^{(1,1)} = 0$$
$$j_{\perp}^{(0,0)} = j_{\perp}^{(1,0)} = j_{\perp}^{(0,1)} = j_{\perp}^{(0,2)} = j_{\perp}^{(1,1)} = 0$$

Now from the general equation for u it is easy to obtain

$$\sum_a \left(\rho_a \frac{d_a \mathbf{u}_a}{dt} + \nabla \cdot \boldsymbol{P}_a \right) = \sigma \mathbf{E} + \frac{1}{c} [j\mathbf{B}]$$

The right side can be transformed in a standard manner using Maxwell's equations, after which the equation takes the form

$$\sum_a \left\{ \rho_a \left(\frac{\partial}{\partial t} + \mathbf{u}_a \cdot \nabla \right) \mathbf{u}_a + \nabla \cdot \boldsymbol{P}_a \right\} = \frac{1}{4\pi} \nabla \cdot \left\{ \mathbf{EE} + \mathbf{BB} - \frac{1}{2} (E^2 + \mathbf{B}^2) \boldsymbol{I} \right\} - \frac{1}{4\pi c} \frac{\partial}{\partial t} [\mathbf{E}_{\perp} \mathbf{B}] \tag{3.16}$$

From this we have the zero-order approximation equation for $\mathbf{V}$

$$\begin{aligned} \rho \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V} + \rho \mathbf{e}_1 \mathbf{V} \mathbf{e}_1 : \nabla \mathbf{V} + \sum_a \rho_a \{ 2u_{\parallel a} \mathbf{e}_1 \cdot \nabla \mathbf{V} - 2u_{\parallel a} \mathbf{e}_1 \mathbf{e}_1 \mathbf{e}_1 : \nabla \mathbf{V} \\ + u_{\parallel a}^2 \mathbf{e}_1 \cdot \nabla \mathbf{e}_1 \} = \mathbf{e}_1 E_{\parallel}^{(1,0)} \sum_a \frac{e_a}{m_a} \rho_a + \sum_a \{ (\boldsymbol{I} - \mathbf{e}_1 \mathbf{e}_1) \cdot \nabla p_{\perp a} \\ + (p_{\parallel a} - p_{\perp a}) \mathbf{e}_1 \cdot \nabla \mathbf{e}_1 \} = \frac{1}{4\pi} \nabla \cdot B^2 \left(\mathbf{e}_1 \mathbf{e}_1 - \frac{1}{2} \boldsymbol{I} \right) \end{aligned} \tag{3.17}$$

where $\rho = \sum_a \rho_a$.

Equations (3.5)-(3.13), (3.15), and (3.17) are the required equations of the zero-order approximation. But they do not form a closed system, since they contain unknown fourth-order moments of $\mathbf{R}$. In principle we can write a general equation for $\mathbf{R}$, but it contains fifth-order moments, etc. We can close the above system by finding an approximate expression for $\mathbf{R}$, for example, using Grad's method of moments [10-12]:

$$\rho \boldsymbol{R} = [\{ p_{\perp} (\boldsymbol{I} - \mathbf{e}_1 \mathbf{e}_1) + p_{\parallel} \mathbf{e}_1 \mathbf{e}_1 \} \{ p_{\perp} (\boldsymbol{I} - \mathbf{e}_1 \mathbf{e}_1) + p_{\parallel} \mathbf{e}\mathbf{e}_1 \}]^s$$

4. We now consider the equations of the first-order approximation in ε and the zero-order approximation in μ (omitting the subscript 0 in expansions in μ). The first-order approximation for the equation of continuity is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho (\mathbf{u}_{\perp} + \mathbf{u}_{\parallel} \mathbf{e}_1) = 0 \tag{4.1}$$

If we introduce the vector $\mathbf{s}$, the "deficient" component of the transverse velocity $\mathbf{u}_{\perp} \equiv \mathbf{V} + \mathbf{s}$, from (2.4) in the first approximation we have

$$\mathbf{s} = \frac{1}{\Omega} \left[\mathbf{e}_1 \frac{d\mathbf{V}}{dt} \right]^{(0)} + \frac{1}{\Omega \rho^{(0)}} \{ [\mathbf{e}_1 \nabla p_{\perp}] + (p_{\parallel} - p_{\perp}) [\mathbf{e}_1 (\mathbf{e}_1 \cdot \nabla) \mathbf{e}_1] \}^{(0)} \tag{4.2}$$

The vector **s** includes the effect of the gradient, centrifugal, and other drift motions of the plasma particles. The equation for $\mathbf{V}^{(1)}$ is obtained from the general equation (3.16)

$$\rho^{(0)} \frac{\partial \mathbf{V}^{(1)}}{\partial t} + \left\{ \sum_a \rho_a \left(\frac{\partial \mathbf{u}_{\perp a}}{\partial t} + u_{\parallel a} \frac{\partial \mathbf{e}_1}{\partial t} + \mathbf{e}_1 \frac{\partial u_{\parallel a}}{\partial t} + \mathbf{u}_a \cdot \nabla \mathbf{u}_a \right) \right\}^{(1)} = \nabla \cdot \left\{ \frac{B^2}{4\pi} \left(\mathbf{e}_1\mathbf{e}_1 - \frac{1}{2} \boldsymbol{I} \right) - \boldsymbol{P} \right\}^{(1)} \tag{4.3}$$

where $\boldsymbol{P} = \sum_a \boldsymbol{P}_a$.

Now there follow the equations

$$\frac{\partial u_{\parallel}^{(1)}}{\partial t} - \left(\mathbf{u}_\perp \cdot \frac{\partial \mathbf{e}_1}{\partial t} \right)^{(1)} + \left\{ \mathbf{e}_1 (\mathbf{V} + u_\parallel \mathbf{e}_1) : \nabla \mathbf{u}_\perp + (\mathbf{V} + u_\parallel \mathbf{e}_1) \cdot \nabla u_\parallel + \mathbf{e}_1 \mathbf{s} : \nabla \mathbf{V} \right\}^{(1)} + \frac{1}{\rho^{(0)}} \left\{ (\nabla \cdot \boldsymbol{P})_\parallel^{(1)} - \frac{\rho^{(1)}}{\rho^{(0)}} (\nabla \cdot \boldsymbol{P})_\parallel^{(0)} \right\} = \frac{e}{m} E_\parallel^{(2)} \tag{4.4}$$

$$\frac{d_0 p_\perp^{(1)}}{dt} + \mathbf{u}^{(1)} \cdot \nabla p_\perp^{(0)} = \left\{ -2p_\perp \nabla \cdot \mathbf{u} + p_\perp \mathbf{e}_1\mathbf{e}_1 : \nabla \mathbf{u} - \nabla \cdot \mathbf{q} + \frac{1}{2} \mathbf{e}_1\mathbf{e}_1 : (\nabla \cdot \boldsymbol{Q}) \right\}^{(1)} - \boldsymbol{\sigma}^{(1)} : \nabla \mathbf{u}^{(0)} - \frac{1}{2} \boldsymbol{\sigma}^{(1)} : \frac{d_0}{dt} (\mathbf{e}_1\mathbf{e}_1)^{(0)} \tag{4.5}$$

$$\frac{d_0 p_\parallel^{(1)}}{dt} + \mathbf{u}^{(1)} \cdot \nabla p_\parallel^{(0)} = \{ -p_\parallel \nabla \cdot \mathbf{u} - 2p_\parallel \mathbf{e}_1\mathbf{e}_1 : \nabla \mathbf{u} - \mathbf{e}_1\mathbf{e}_1 : (\nabla \cdot \boldsymbol{Q}) \}^{(1)} - 2\mathbf{e}_1^{(0)} (\mathbf{e}_1^{(0)} \cdot \boldsymbol{\sigma}^{(1)}) : \nabla \mathbf{u}^{(0)} + \boldsymbol{\sigma}^{(1)} : \frac{d_0}{dt} (\mathbf{e}_1\mathbf{e}_1)^{(0)} \tag{4.6}$$

$$\frac{\partial \mathbf{e}_1^{(1)}}{\partial t} = \{ \mathbf{e}_1 \cdot \nabla \mathbf{V} - \mathbf{V} \cdot \nabla \mathbf{e}_1 - \mathbf{e}_1\mathbf{e}_1\mathbf{e}_1 : \nabla \mathbf{V} \}^{(1)} - \frac{E_\parallel^{(1)}}{B} \mathrm{rot}_\perp \mathbf{e}_1^{(0)} = \frac{1}{B} [\mathbf{e}_1^{(0)} \nabla E_\parallel^{(1)}] \tag{4.7}$$

$$E_\parallel^{(2)} \sum_a \frac{e_a^2 n_a^{(0)}}{m_a} + E_\parallel^{(1)} \sum_a \frac{e_a^2 n_a^{(1)}}{m_a} = \sum_a e_a \left\{ u_{\parallel a} \nabla \cdot n_a \mathbf{u}_a + \frac{1}{m_a} \mathbf{e}_1 \nabla : \boldsymbol{P}_a \right\}^{(1)} \tag{4.8}$$

$$\begin{aligned} \rho^{(0)} \Big\{ & \frac{d_0 2q_\parallel^{\parallel(1)}}{dt} + \mathbf{u}^{(1)} \cdot \nabla q_\parallel^{\parallel(0)} + [2q_\parallel^\parallel \nabla \cdot \mathbf{u} + \mathbf{e}_1\mathbf{e}_1 : (\mathbf{e}_1 \nabla : \boldsymbol{R}) + 6q_\parallel^\parallel \mathbf{e}_1\mathbf{e}_1 : \nabla \mathbf{u}]^{(1)} - 3Q_{11\alpha}^{(1)} \left(\mathbf{e}_\alpha \cdot \frac{d\mathbf{e}_1}{dt} - \mathbf{e}_1\mathbf{e}_\alpha : \nabla \mathbf{u} \right)^{(0)} \Big\} \\ & + 3 \left(\frac{\rho^{(1)} p_\parallel^{(0)}}{\rho^{(0)}} - p_\parallel^{(1)} \right) \{ \mathbf{e}_1 \cdot \nabla p_\parallel + (p_\parallel - p_\perp) \nabla \cdot \mathbf{e}_1 \}^{(0)} = 3p_\parallel^{(0)} \{ \mathbf{e}_\alpha \cdot \nabla P_{\alpha 1} + P_{1\alpha} \nabla \cdot \mathbf{e}_\alpha + P_{\alpha\beta} \mathbf{e}_1\mathbf{e}_\alpha : \nabla \mathbf{e}_\beta \}^{(1)} \end{aligned} \tag{4.9}$$

$$\begin{aligned} \rho^{(0)} \Big\{ & \frac{d_0 2q_\parallel^{\perp(1)}}{dt} + 2\mathbf{u}^{(1)} \cdot \nabla q_\parallel^{\perp(0)} + [2q_\parallel^\perp (\boldsymbol{I} + \mathbf{e}_1\mathbf{e}_1) : \nabla \mathbf{u} + (\boldsymbol{I} - \mathbf{e}_1\mathbf{e}_1) : (\mathbf{e}_1 \nabla : \boldsymbol{R}) \\ & + 2 (Q_{122}\mathbf{e}_2\mathbf{e}_2 + Q_{133}\mathbf{e}_3\mathbf{e}_3) : \nabla \mathbf{u}]^{(1)} + 3Q_{11\alpha}^{(1)} \mathbf{e}_\alpha \cdot \frac{d_0 \mathbf{e}_1}{dt} \\ & - Q_{\alpha\alpha\beta}^{(1)} \mathbf{e}_\beta \cdot \frac{d_0 \mathbf{e}_1}{dt} + [2Q_{112}^{(1)} \mathbf{e}_2\mathbf{e}_1 + 2Q_{113}^{(1)} \mathbf{e}_3\mathbf{e}_1 + 2Q_{123}^{(1)} (\mathbf{e}_2\mathbf{e}_3 + \mathbf{e}_3\mathbf{e}_2) \\ & + (Q_{222}^{(1)} + Q_{233}^{(1)}) \mathbf{e}_1\mathbf{e}_2 + (Q_{223}^{(1)} + Q_{333}^{(1)}) \mathbf{e}_1\mathbf{e}_3] : \nabla \mathbf{u}^{(0)} \Big\} + \frac{2p_\perp^{(0)} \rho^{(1)}}{\rho^{(0)}} \{ \mathbf{e}_1 \cdot \nabla p_\parallel \\ & + (p_\parallel - p_\perp) \nabla \cdot \mathbf{e}_1 \}^{(0)} = \{ 2P_{12} (\mathbf{e}_\alpha \cdot \nabla P_{2\alpha} + P_{2\alpha} \nabla \cdot \mathbf{e}_\alpha + P_{\alpha\beta} \mathbf{e}_2\mathbf{e}_\beta : \nabla \mathbf{e}_\alpha) \\ & + 2P_{13} (\mathbf{e}_\alpha \cdot \nabla P_{3\alpha} + P_{3\alpha} \nabla \cdot \mathbf{e}_\alpha + P_{\alpha\beta} \mathbf{e}_3\mathbf{e}_\beta : \nabla \mathbf{e}_\alpha) \\ & + (P_{22} + P_{33}) (\mathbf{e}_\alpha \cdot \nabla P_{\alpha 1} + P_{1\alpha} \nabla \cdot \mathbf{e}_\alpha + P_{\alpha\beta} \mathbf{e}_1\mathbf{e}_\beta : \nabla \mathbf{e}_\alpha) \}^{(1)}. \end{aligned} \tag{4.10}$$

Now it remains to determine the tensors $\mathbf{P}^{(1)}$ and $\mathbf{Q}^{(1)}$ which describe the viscosity and thermal conductivity of a collisionless plasma with a strong magnetic field. To find $\mathbf{P}^{(1)}$ we have to use the first approximation equation (2.5):

$$([\boldsymbol{P} \times \mathbf{e}_1]^s)^{(1)} = \frac{1}{\Omega} \left(\frac{d\boldsymbol{P}}{dt} + \boldsymbol{P} \nabla \cdot \mathbf{u} + \nabla \cdot \boldsymbol{Q} + [\boldsymbol{P} \cdot \nabla \mathbf{u}]^s \right)^{(0)}$$

From this, after scalar multiplication by the appropriate dyads, constructed from the unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$, there follow:

$$P_{13}^{(1)} = \frac{1}{\Omega} \left\{ (p_\parallel \mathbf{e}_2\mathbf{e}_1 + p_\perp \mathbf{e}_1\mathbf{e}_2) : \nabla \mathbf{u} + (p_\parallel - p_\perp) \left(\mathbf{e}_2 \cdot \frac{\partial \mathbf{e}_1}{\partial t} + \mathbf{e}_2 \mathbf{u} : \nabla \mathbf{e}_1 \right) + \mathbf{e}_2 \cdot \nabla q_\parallel^\perp + 2 (q_\parallel^\parallel - q_\parallel^\perp) \mathbf{e}_2\mathbf{e}_1 : \nabla \mathbf{e}_1 \right\}^{(0)} \tag{4.11}$$

$$P_{12}^{(1)} = -\frac{1}{\Omega} \left\{ (p_\parallel \mathbf{e}_3\mathbf{e}_1 + p_\perp \mathbf{e}_1\mathbf{e}_3) : \nabla \mathbf{u} + (p_\parallel - p_\perp) \left(\mathbf{e}_3 \cdot \frac{\partial \mathbf{e}_1}{\partial t} + \mathbf{e}_3 \mathbf{u} : \nabla \mathbf{e}_1 \right) + \mathbf{e}_3 \cdot \nabla q_\parallel^\perp + 2 (q_\parallel^\parallel - q_\parallel^\perp) \mathbf{e}_3\mathbf{e}_1 : \nabla \mathbf{e}_1 \right\}^{(0)} \tag{4.12}$$

$$P_{23}^{(1)} = \frac{1}{2\Omega} (\mathbf{e}_2\mathbf{e}_2 - \mathbf{e}_3\mathbf{e}_3) : (p_\perp \nabla \mathbf{u} + q_\parallel^\perp \nabla \mathbf{e}_1)^{(0)} \tag{4.13}$$

$$P_{33}^{(1)} = -P_{22}^{(1)} = p_\perp^{(1)} + \frac{1}{2\Omega} (\mathbf{e}_2\mathbf{e}_3 + \mathbf{e}_3\mathbf{e}_2) : (p_\perp \nabla \mathbf{u} + q_\parallel^\perp \nabla \mathbf{e}_1)^{(0)} \tag{4.14}$$

$$P_{\parallel}^{(1)} = p_{\parallel}^{(1)} \tag{4.15}$$

If in these equations we replace $\partial e_1/\partial t$ in accordance with (3.13), it is easy to obtain Macmahon's equations [4] (cf. [12]).

The tensor $\mathbf{Q}^{(1)}$ is determined by Eq. (2.8)

$$([\boldsymbol{Q} \times \mathbf{e}_1]^s)^{(1)} = \frac{1}{\Omega}\left(\frac{d\boldsymbol{Q}}{dt} + \boldsymbol{Q}\nabla\cdot\mathbf{u} + \nabla\cdot\boldsymbol{R} + [\boldsymbol{Q}\cdot\nabla\mathbf{u}]^s - \frac{1}{\rho}[\boldsymbol{P}\nabla\cdot\boldsymbol{P}]^s\right)^{(0)}$$

From this we obtain (cf. [4])

$$Q_{113}^{(1)} = \frac{1}{\Omega}\Big\{2(q_{\parallel}^{\parallel} - q_{\parallel}^{\perp})\left(\mathbf{e}_2\cdot\frac{\partial \mathbf{e}_1}{\partial t} + \mathbf{e}_2\mathbf{u} : \nabla\mathbf{e}_1\right) + 2(q_{\parallel}^{\parallel}\mathbf{e}_2\mathbf{e}_1 + q_{\parallel}^{\perp}\mathbf{e}_1\mathbf{e}_2) : \nabla\mathbf{u}$$
$$+ (R_1 - 3R_2)\mathbf{e}_2\mathbf{e}_1 : \nabla\mathbf{e}_1 + \mathbf{e}_2\cdot\nabla R_2 - \frac{p_{\parallel}}{\rho}[\mathbf{e}_2\cdot\nabla p_{\perp} + (p_{\parallel} - p_{\perp})\mathbf{e}_2\mathbf{e}_1 : \nabla\mathbf{e}_1]\Big\}^{(0)} \tag{4.16}$$

$$Q_{112}^{(1)} = -\frac{1}{\Omega}\Big\{2(q_{\parallel}^{\parallel} - q_{\parallel}^{\perp})\left(\mathbf{e}_3\cdot\frac{\partial \mathbf{e}_1}{\partial t} + \mathbf{e}_3\mathbf{u} : \nabla\mathbf{e}_1\right) + 2(q_{\parallel}^{\parallel}\mathbf{e}_3\mathbf{e}_1 + q_{\parallel}^{\perp}\mathbf{e}_1\mathbf{e}_3) : \nabla\mathbf{u}$$
$$+ (R_1 - 3R_2)\mathbf{e}_3\mathbf{e}_1 : \nabla\mathbf{e}_1 + \mathbf{e}_3\cdot\nabla R_2 - \frac{p_{\parallel}}{\rho}[\mathbf{e}_3\cdot\nabla p_{\perp} + (p_{\parallel} - p_{\perp})\mathbf{e}_3\mathbf{e}_1 : \nabla\mathbf{e}_1]\Big\}^{(0)} \tag{4.17}$$

$$Q_{123}^{(1)} = \frac{1}{2\Omega}(\mathbf{e}_2\mathbf{e}_2 - \mathbf{e}_3\mathbf{e}_3) : \left\{q_{\parallel}^{\perp}\nabla\mathbf{u} + \left(R_2 - \frac{1}{4}R_3\right)\nabla\mathbf{e}_1\right\}^{(0)} \tag{4.18}$$

$$(Q_{133} - Q_{122})^{(1)} = \frac{1}{\Omega}(\mathbf{e}_2\mathbf{e}_3 + \mathbf{e}_3\mathbf{e}_2) : \left\{q_{\parallel}^{\perp}\nabla\mathbf{u} + \left(R_2 - \frac{1}{4}R_3\right)\nabla\mathbf{e}_1\right\}^{(0)} \tag{4.19}$$

$$-Q_{233}^{(1)} = 2Q_{233}^{(1)} - Q_{222}^{(1)} = \frac{1}{\Omega}\Big\{2q_{\parallel}^{\perp}\mathbf{e}_3\mathbf{e}_1 : \nabla\mathbf{u} + \left(R_2 - \frac{1}{4}R_3\right)\mathbf{e}_3\mathbf{e}_1 : \nabla\mathbf{e}_1$$
$$+ \frac{1}{4}\mathbf{e}_3\cdot\nabla R_3 - \frac{p_{\perp}}{\rho}[\mathbf{e}_3\cdot\nabla p_{\perp} + (p_{\parallel} - p_{\perp})\mathbf{e}_3\mathbf{e}_1 : \nabla\mathbf{e}_1]\Big\}^{(0)} \tag{4.20}$$

$$Q_{223}^{(1)} = Q_{333}^{(1)} - 2Q_{322}^{(1)} = \frac{1}{\Omega}\Big\{2q_{\parallel}^{\perp}\mathbf{e}_2\mathbf{e}_1 : \nabla\mathbf{u} + \left(R_2 - \frac{1}{4}R_3\right)\mathbf{e}_2\mathbf{e}_1 : \nabla\mathbf{e}_1$$
$$+ \frac{1}{4}\mathbf{e}_2\cdot\nabla R_3 - \frac{p_{\perp}}{\rho}[\mathbf{e}_2\cdot\nabla p_{\perp} + (p_{\parallel} - p_{\perp})\mathbf{e}_2\mathbf{e}_1 : \nabla\mathbf{e}_1]\Big\}^{(0)} \tag{4.21}$$

The components $Q_{111}^{(1)} = 2q_{\parallel}^{\parallel}$ and $(Q_{122} + Q_{133})_{(1)} = 2q_{\parallel}^{\perp}$ are described by Eqs. (4.9) and (4.10). Equations (4.11) and (4.21) give approximations of the Navier-Stokes type for the description of a collisionless plasma in a strong magnetic field.

5. By two-dimensional theory we mean the special case when the lines of force of the magnetic field are straight and remain straight as time passes. We only consider phenomena occurring in the plane perpendicular to the lines of force, i.e., we assume that

$$\mathbf{e}_1\cdot\nabla = 0, \quad \nabla\mathbf{e}_1 = 0, \quad u_{\parallel} = 0 \tag{5.1}$$

Then in the zero-order approximation we obtain a quite simple series of equations

$$D\rho_a = -\rho_a\nabla\cdot\mathbf{V} \tag{5.2}$$

$$Dp_{\perp a} = -2p_{\perp a}\nabla\cdot\mathbf{V} \tag{5.3}$$

$$DB = -B\nabla\cdot\mathbf{V} \tag{5.4}$$

$$\rho D\mathbf{V} = -\nabla\left(p_{\perp} + \frac{1}{8\pi}B^2\right) \tag{5.5}$$

where

$$D \equiv \frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla, \quad \rho = \sum_a \rho_a, \quad p_{\perp} = \sum_a p_{\perp a}$$

To an accuracy of order ε^2, μ the equations of two-dimensional theory have the form

$$\frac{\partial\rho_a}{\partial t} + \nabla\cdot\rho_a\mathbf{V} + \frac{1}{\Omega_a}\mathbf{e}_1\cdot\left\{\left[\nabla\frac{\rho_a}{\rho}\nabla\left(p_{\perp} + \frac{B^2}{8\pi}\right)\right] - \left[\left\{\nabla p_{\perp a} - \frac{\rho_a}{\rho}\nabla p_{\perp}\right\}\nabla\ln B\right]\right\}^{(0)} = 0$$

$$\frac{\partial p_{\perp a}}{\partial t} + \mathbf{V}\cdot\nabla p_{\perp a} + 2p_{\perp a}\nabla\cdot\mathbf{V} + \frac{1}{\Omega_a}\left\{\frac{1}{\rho}\left[\nabla p_{\perp a}\nabla\left(p_\perp + \frac{B^2}{8\pi}\right)\right]'\right.$$

$$\left. + \frac{2p_{\perp a}}{\rho}\mathbf{e}_1\cdot\left(\left[\nabla\left(p_\perp + \frac{B^2}{8\pi}\right)\nabla\ln\rho\right] + [\nabla p_\perp\nabla\ln B]\right) - \frac{1}{2}\mathbf{e}_1\cdot[\nabla R_{3a}\nabla\ln B]\right\}^{(0)} = 0$$

$$\rho D\mathbf{V} + \nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \sum_a \frac{1}{\Omega_a}\left(\left[\mathbf{e}_1\left\{\nabla p_{\perp a} - \frac{\rho_a}{\rho}\nabla\left(p_\perp + \frac{B^2}{8\pi}\right)\right\}\right]\cdot\nabla\mathbf{V}\right.$$

$$- 2\left\{p_{\perp a} - \frac{\rho_a}{\rho}\left(p_\perp + \frac{B^2}{8\pi}\right)\right\}[\mathbf{e}_1\nabla]\,\nabla\cdot\mathbf{V} - [\mathbf{e}_1\nabla]\underset{\llcorner\uparrow}{\mathbf{V}}\cdot\left\{\nabla p_{\perp a} - \frac{\rho_a}{\rho}\nabla\left(p_\perp + \frac{B^2}{8\pi}\right)\right\}$$

$$\left. + \left\{\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) - \left[\mathbf{e}_1\nabla\left(p_\perp + \frac{B^2}{8\pi}\right)\right]\right\}\mathbf{V}\cdot\nabla\frac{\rho_a}{\rho}\right)^{(0)} + \nabla\cdot\boldsymbol{\sigma}^{(1)} = 0$$

$$\frac{\partial B}{\partial t} = -\nabla\cdot B\mathbf{V}$$

The symbol $\llcorner\uparrow$ indicates that the operator ∇ acts only on **V**.

The system is not closed until we have determined $R_{3a}^{(0)}$. From Grad's method of moments [12] we can write

$$R_{3a}^{(0)} = 4p_\perp/\rho$$

Then the above system of equations is closed. Using (4.11)-(4.15) it is easy to find that in two-dimensional theory

$$\nabla\cdot\boldsymbol{\sigma}_a^{(1)} = \frac{p_{\perp a}}{2\Omega_a}\left\{\Delta_\perp[\mathbf{e}_1\mathbf{V}] + \nabla\ln\frac{p_{\perp a}}{B}\cdot\nabla[\mathbf{e}_1\mathbf{V}] - \left[\mathbf{e}_1\nabla\ln\frac{p_{\perp a}}{B}\right]\cdot\nabla\mathbf{V}\right\}$$

From this, in particular, there follow the familiar equations [13] for the components of $\nabla\cdot\sigma^{(1)}$ in a cylindrical coordinate system.

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